Unit - II

Thermal Sensors

Introduction :

Thermal sensors are primarily temperature sensors, also called as thermodynamic sensors and these sensors are classified into two types namely primary and secondary temperature sensors. They are :

Primary Sensors

1. Gas thermometer
2. Vapour pressure type
3. Acoustic type
4. Refractive Index thermometer
5. Dielectric constant type
6. He low temperature thermometer
7. Total radiation and spectral radiation type
8. Magnetic type
9. Nuclear orientation type
10. Noise type

Secondary sensors.

1. Thermal expansion types: Solid liquid and gas.
2. Resistance thermometer
3. Thermoemf type
4. Diodes, transistors or junction Semiconductor types
5. Adapted radiation type
6. Quartz crystal thermometer
7. NQR thermometer
8. Ultrasonic type.

There are different kinds of heat flux sensors which measure heat flux in terms of temparature difference, like pneumatic type, pyroelectric type and so on. The main physical quantity, Q is usually expressed in terms of magnitude N and its units. U,

$$Q = NU$$

* Gas Thermometric Sensors :

These sensors are based on the gas low.

$$PV = nRT$$

Where

P = the Pressure,

V = Volume of the gas,

R = the gas constant

T = the temperature in K-scale, and

n = the number of moles

The above relation is true for all ideal gases and is approximately true for real gases at low pressures. For a real gas, the above relation becomes.

$$PV = nRT\left[1 + \beta_1(T)\left(\frac{n}{V}\right) + \beta_2(T)\left(\frac{n}{V}\right)^2 + \cdots\right]$$

Where β_1's are different functions of temperatures, gas thermometers can be of two different types, they are

1) Constant volume thermometers where P is proportional to T

2) Constant pressure thermometers where $\bar{V}$ is proportional to T.

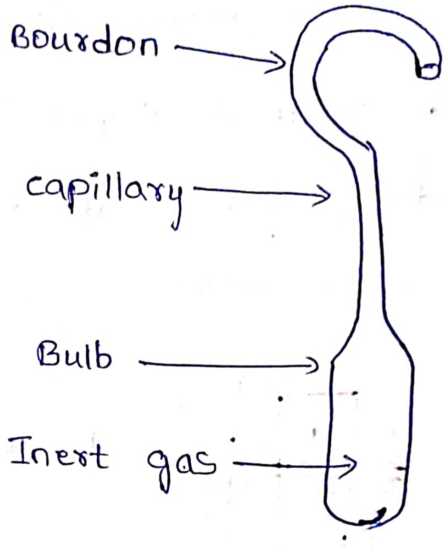
Here the contributions of higher order terms become larger at lower temperatures and higher pressures. So the above relation becomes.

$$PV = nRT\left[1 + \beta_1(T)\left(\frac{P}{RT}\right)\right]$$

If the pressure and temperature P_r and T_r are known as reference conditions, then, the equation becomes

$$T = T_r\left(\frac{P}{P_r}\right)\left[\frac{1 + \beta_1(T_r)\left(\frac{P_r}{RT_r}\right)}{1 + \beta_1(T)\left(\frac{P}{RT}\right)}\right]$$

A consequence of the gas pressure thermometer is the vapour pressure thermometer, in this a suitable liquid is filled in a bulb and keeping enough space above the surface of the liquid for vapour pressure to form to be saturated at all temparatures, with increase in temperature, the above pressure increases according to claussius - clapeyron equation.

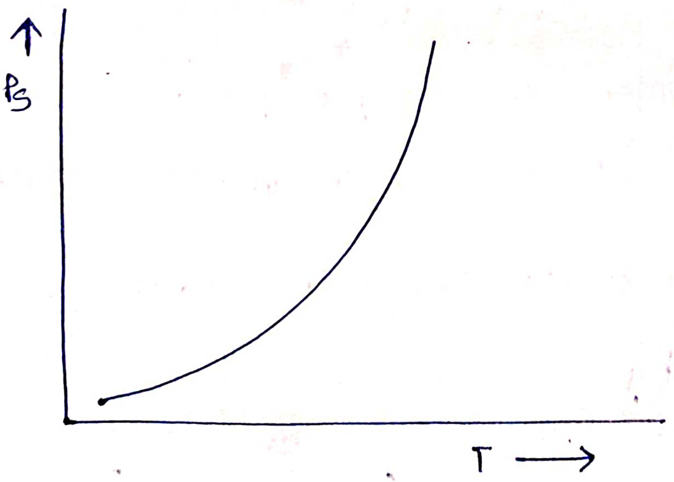


$$T \frac{dP_s}{dT} = \frac{H_v}{V_g - V_l}$$

Where

P_s = saturated vapour pressure
H_v = molar heat of vaporization
V_g = molar volume in gaseous state
V_l = molar volume in liquid state.

The P_s versus T curves for different liquids are shown as



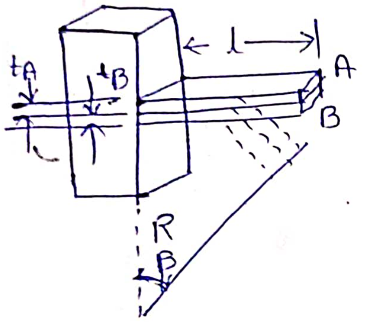
Liquid	Range (°c)
Methyl alcohol	0 - 50
n - Butane	20 - 80
Methyl Bromide	30 - 85
Ethyl chloride	30 - 100
Ethyl ether	60 - 160
Ethyl alcohol	30 - 180
Toluene	150 - 250

vapour pressure variation with temperature.

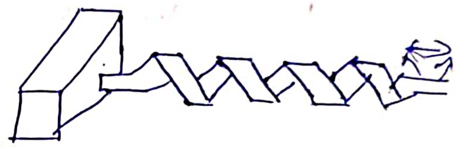
* Thermal Expansion type of thermometric Sensors.

The earliest of this kind is the solid expansion type bimetallic sensor which uses the difference in thermal expansion coefficients of the different metals used.

Two metal strips A and B of thickness t_A and t_B and thermal expansion coefficients α_A and α_B are firmly bonded together at a temperature, usually lower than the reference value.



Cantilever type bimetal thermometer



Helix type bimetal thermometer

When the temperature of the cantilever or the helix is raised by heating or lowered by cooling, one strip expands or contracts more and free end of either of the two moves as shown.

The cantilever bends into a circular arc with radius of curvature R and is given by

$$R = \frac{(t_A+t_B)\left[3\left(1+\frac{t_B}{t_A}\right)^2+\left(1+\left(\frac{t_B}{t_A}\right)\left(\frac{Y_B}{Y_A}\right)\right)\left\{\left(\frac{t_B}{t_A}\right)^2+\frac{t_A Y_A}{t_B Y_B}\right\}\right]}{6(\alpha_A-\alpha_B)(T_h-T_b)\left[1+\frac{t_B}{t_A}\right]^2}$$

Where Y is the Young's modulus,
T_h is the raised temperature, and
T_b is the bonding temperature

The above equation is simplified using $t_A = t_B = t$ and $Y_A \approx Y_B$
This gives

$$R = \frac{4t}{3(\alpha_A - \alpha_B)(T_h - T_b)}$$

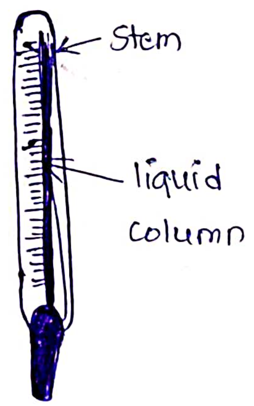
The angular deflection β per unit temperature change, that is sensitivity (for small β) is given by.

$$S_T^{\beta} = \frac{\beta}{(T_h - T_b)} = 3l \frac{\alpha_A - \alpha_B}{4t}$$

l is length of cantilever

S_T^{β} increases linearly with length and inversly with strip thickness for a given pair of metal elements.

The next type of sensor is the liquid-in-glass thermometer the liquid in majority of the cases being mercury. It utilizes expansion property of the liquid kept in the bulb to which a capillary, closed at the far end, is attached in



which the expanded liquid rises and an indication in mm, calibrated directly in temperature scale, is obtained. The range of a clinical thermometer is $-35°c$ to $300°c$ and the upper limit is $357°c$, its boiling point.

Acoustic Temperature Sensor :

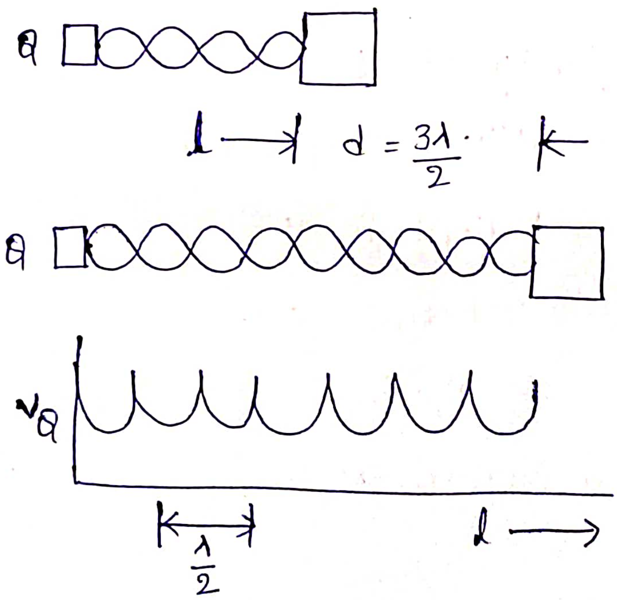
When a longitudinal (acoustic) wave propagates through an ideal gas, it has a speed C_i is given by

$$C_i = \left(\frac{\gamma RT}{M}\right)^{1/2}$$

Where M is the molecular weight of the gas and $\gamma = C_p/C_v$ is the ratio of specific heats ($\gamma = 5/3$ for monoatomic gases), then measuring temperature T can be given by.

$$T = \frac{MC_i^2}{\gamma R}.$$

This sensor is made in acoustic heliom interferometer, in which a quartz crystal excited to its resonance frequency is used to transmit this wave through a gas (He) column, to be faced by the Piston. The wave is reflected at the piston surface to form a pattern as shown.

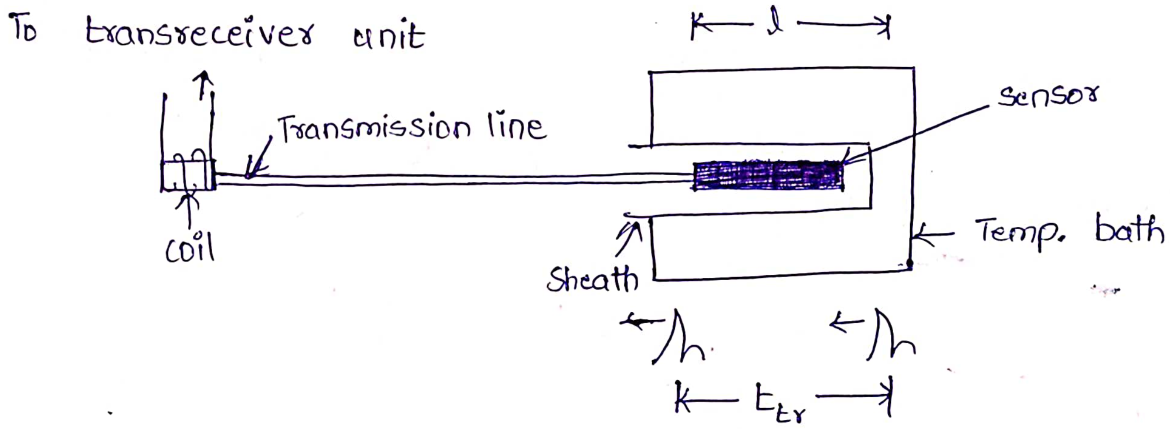


When the path length l has a multiple number of half-wavelengths and correspondingly the gas column is set resonate at each

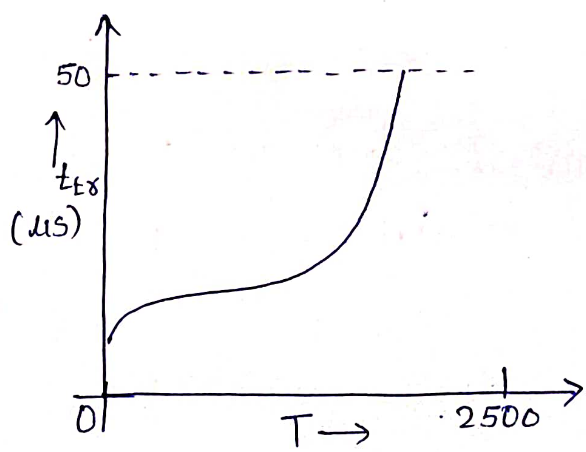
Such half-wavelength gap, with the piston moving away from the crystal at each resonant peak, the crystal gives out max energy and hence the voltage V_Q across the piston is varied.

* There is a non resonant acoustic sensor that utilizes the pulse-echo transmit time difference which changes with the temperature. An ultrasonic pulse is transmitted through the sensor, a part of which is reflected at the entrance and a part at the end.

* The reflected pulses are received by the transceiver coil at an interval of t_{tr} called the tranit time. The pulse that travels the entire length of the sensor is delayed morelless depending on the change in the sensor temperature.



The nature of plot between t_{tr} and temperature T is



Transit Time versus temperature plot

Dielectric constant and Refractive Index Thermosensors:

This kind of sensors are used to measure the gas temperature, and these are developed based on two Relations;

(i) Clausius - Mossotti relation and

(ii) The relation between refractive index μ and dielectric constant χ of a gas.

→ The clausius - Mossotti relation is valid for an ideal gas and is given by

$$\frac{\chi - 1}{\chi + 2} = M_\chi \frac{n}{V}$$

χ = dielectric constant
M_χ = Molecular Polarizability

by the equation $PV = nRT$, the above equation becomes

$$T = \frac{(\chi+2) M_\chi}{(\chi-1) R} P$$

→ For real gases, however, virial expansion of the dielectric constant has to be taken into account and 'extrapolation' technique is done.

Refractive index thermometer uses the relation

$$\frac{\mu^2 - 1}{\mu + 2} = \frac{M_\chi n}{V} = M_\chi \frac{P}{RT}$$

→ For non-ideal gases, a virial expansion of $\left(\frac{\mu^2-1}{\mu^2+2}\right)$ is used.

Heliom Low Temperature Thermometer :

This thermometer is developed based on pressure - temperature equilibrium relationship for ^3He enclosed in a constant volume chamber provided on one side with a Be-Cu diaghragm on which a strain guage is bonded for measuring pressure of expanded He at low temperature where it can exist in liquid - Solid equilibrium state at a minimum temperature of $0.32^\circ K$.

Nuclear Thermometer :

This thermometer works on a basic working principle of a temperature dependent nuclear characteristic which states that the directional distribution of emission of β or γ radiations from a radioactive nucleus is dependent on the degree of ordering of a of nuclear spins.

This type of thermometer can be used at very low temperatures below 0.1 K and can be designed with and without a magnetic field. The nuclear spins are dependent on temperature with Boltzmann distribution., the relative population P_m is given by

$$P_m = \frac{\exp(-E_m/KT)}{\sum_n \exp(-E_n/KT)}$$

Where E_n are the energies of nuclear hypefine states, n taking the values of quantum numbers.

In thermodynamic equilibrium, the spin system temperature equals the lattice temperature., if T is very large, all P_m's are equal and the radiation is said to be emitted isotropically.

However if T is very small, $T \leq \frac{E_n}{K}$, P_m's have different values and an anisotropic emission occurs resulting in difference in nuclear orientation about the axis of quantization. of the nuclear spin system. The radiation emitted is in a three dimensional 'field' with intensity, direction and temperature as coordinates.

In practical sensors, suitable radioactive nuclei are incorporated in a 'host lattice' which usually is a ferro-magnetic in nature. The nuclei used here are ^{54}Mn in Fe, Ni, Cu, Zn or ^{60}Co in Fe, Co, and Ni with known decay schemes

The Normalized direction distribution (NDD) with specified angles (α_s) are calulated to a function of T. when T is very Large.

Magnetic Thermometer :

Another low temperature thermal sensor is based on the change of magnetic susceptibility χ of a paramagnetic substance (salt). The materials used for different temperature ranges are

Cerium Magnesium Nitrate (CMN) - 0.01 - 2.5 °K

Chromic Methylammonium Alum (CMA) - 0.3 - 30°K.

Manganous Ammonium Sulphate (MAS) and Gadolinium Sulphate (GS) - 0.9 - 80 °K

$$\chi = \chi_0 + \frac{C}{T + \bar{\theta} + \frac{\alpha}{T}}$$

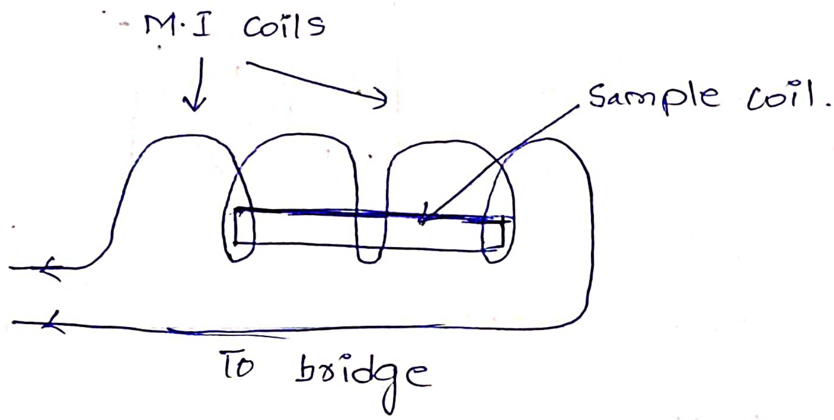
χ_0 is the temperature independent susceptibility

$\bar{\theta}$: dipole coupling of local field & ion-exchange interaction

$\frac{\alpha}{T}$: Stark splitting of ground state with field.

Both $\bar{\theta}$ and $\frac{\alpha}{T}$ are dependent on same geometry and on the angle between the applied field and the sample crystal axis. For a spherical sample, for example, dipole coupling of the local field vanishes. Measurement is made by putting the sample crystal in a pair of mutually coupled coils. which are connected to one arm of a measuring bridge for measuring the change in mutual Inductance M.

$$M = M_0 + \frac{B}{T + \bar{\theta} + \frac{\alpha}{T}}$$



In practice, M_0 and B depend upon χ_0, c and coil design, $\bar{\theta}$ is a salt property and also depends on same shape. and α is only dependent on property of the salt.

For CMN, both $\bar{\theta}$ and α very small and the equation becomes

$$M = M_0 + \frac{B}{T}.$$

It is required that the four constants M_0, B, $\bar{\theta}$ and α are evaluated by calibrating at four temperatures and the calibration is necessary for different samples individually.

* Resistance change type Thermometric sensors.

This type of sensors are works on a basic principle that temperature is dependent on electrical conduction in conductors and semiconductors.

The resistance changes (ΔR) with change in temperature ΔT, ΔR is measured by electrical circuits and indicating systems. Resistive nature is observed due to phonon absorption/emission, is only temperature-dependent and variations in temperature can be caused by the conductivity of the metals. This can be explained by

$$\tau_{r0} = \left[\sum_{i=1}^{n} \left(\frac{1}{\tau_{ri}} \right) \right]^{-1}$$

τ_{ri} = initial relaxation time

τ_{r0} = overall relaxation time.

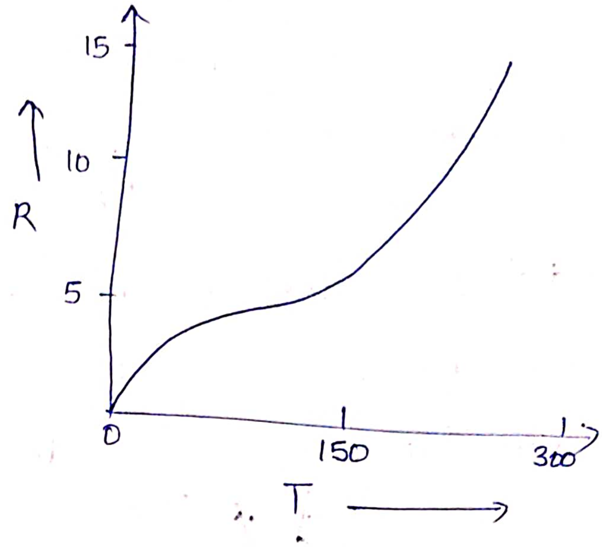
For simple metals such as sodium and potassium, resistance is a simple function of T as long as $T > T_c$, T_c is a characteristic temperature of the metal related to Debye temperature θ_d.

At low temperatures, the relation with T may be positive for some metals (Ni) and negative for some metals (Pt). Increasing temperatures. notices the E_v which is the energy of formation of lattice vacancy. then Resistance becomes.

$$\partial \rho \propto \exp\left(\frac{-E_v}{KT} \right)$$

The nature of the Resistance - Temperature (R-T) for a Sample alloy is shown in graph.

Resistance decreases monotonically with lowering of temperature without a minimum at a Specific value.



Ranges of such semiconducting sensors are generally limited to 300 - 400°C. At very low temperatures, below -260°C conduction is by electron jumps but still the resistance variation with temperature remains exponential.

* Metal Resistance Thermometric Sensors :

In metals, conduction due to scattering of electrons, this scattering leads to the resistivity, and the relation is

$$\rho = \rho_0 \left[1 + \frac{m}{n_e e^2} \left(\frac{1}{\tau_r} \right) \right]$$

which for a specific resistance element as a sensor can be transformed into

$$R = R_0 \left[1 + \sum_{i=1}^{n} \alpha_i (\Delta T)^i \right]$$

The per unit resistance change from the initial value R_0 is given by

$$\frac{R - R_0}{R_0} = \alpha_i \Delta T$$

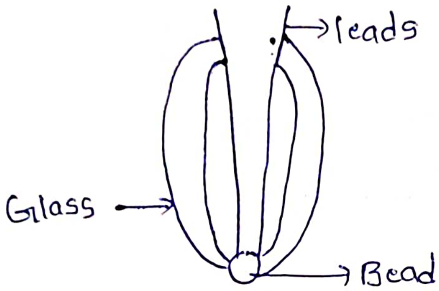
The values of ρ and α_i depend on the purity of materials.

* Thermistors :

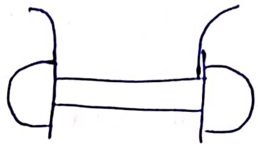
These type of sensors are developed by using semiconductor resistance thermometric sensors made from oxides of metals of transition group

$$R_t = R_0 \exp\beta\left[\frac{1}{T} - \frac{1}{T_0}\right]$$

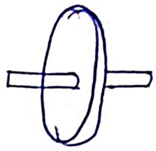
NTC thermistors are named as bead type, rod type, disc type. The operating range of -100 - 300°C. Two platinum wires are stretched apart to a reasonable distance.



Bead type



Rod type



disc type.

The thermistor's characteristics are

1) Resistance value
2) Temperature coefficient of resistance
3) Response time.

Resistivity is usually kept between $100 - 10^6\ \Omega cm$ and resistance values between $5-50\Omega$. Within a working range, a change of R by 30% is preferred. Its coefficient of change in resistance is given by

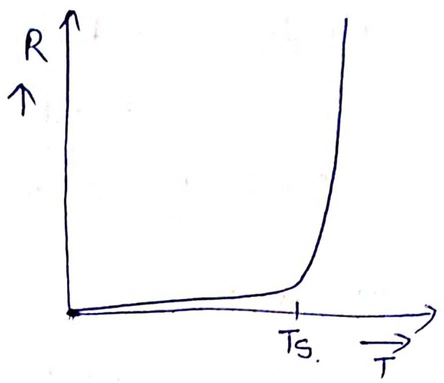
$$\alpha_{th} = \frac{1}{R}\frac{dR}{dT} = \frac{-\beta}{T^2}$$

A thermistor of bead type with 0.2 cm diameter can have a response time as large as 15 s in still condition. The time constant of a thermistor is calculated as

$$\tau = \frac{mC}{hA}$$

Positive temperature coefficient thermistors show large and sudden resistance changes at a temperature called switching or transition temperature T_s as shown.

The switching temperature is dependent on $\frac{Ba}{Sr}$ ratio. By proper proportion T_s may be varied from 15 - 115°C and Such transducers are used as heat switches.



Resistance thermometers need to have a current passing through them which is likely to cause an error often termed as self-heating error. The heat produced in sensor because of this current flows.

(i) Towards the zone whose temperature is to be measured through the surrounding walls and sheaths

(ii) along the leads to a certain extent.

The self-heating error t_h is

$$t_h = \frac{I^2 R}{x_m + x_1}$$

If R_1 and R_2 are resistancens when the currents are I_1 and I_2 then the temperatures are t_1 and t_2

$$t_h = t_M - t_I = \frac{(R_2 - R_1) I^2}{S_{th} R (I_2^2 - I_1^2)}$$

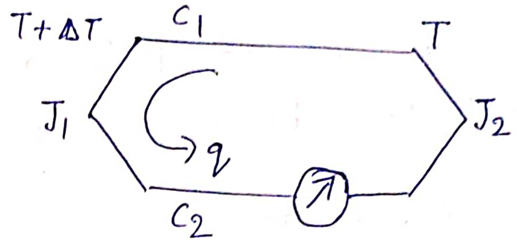
S_{th} is thermal sensitivity of resistance themoter

$$t_I = t_1 - \frac{(t_1 - t_2) I_1^2}{I_1^2 - I_2^2}$$

* Thermoemf sensors :

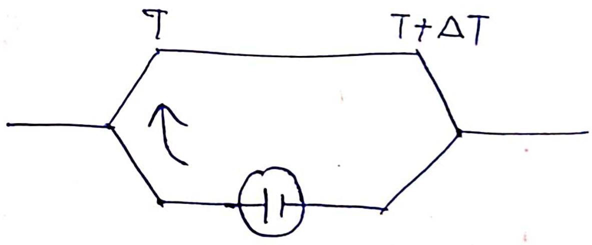
These sensors are thermocouples which are most extensively used in industry, over a wide range of temperatures, This measurements does not require separate supply. A resolution of 0.1 to 0.2°C with increase of about ±5°C.

Two conductors C_1 and C_2 of different compositions are made up into a closed circuit, a small current flows through it if one of the junctions J_1 has a different temperature than the other junction J_2 This current is driven as an emf is generated between these two junctions because of temperature difference (ΔT).



Basic thermocouple

If a charge q passes around the couple in a anti-clock direction at $T+\Delta T$, then the heat absorbed at $T+\Delta T$ is $q\pi_1$ and the heat released at T. is $q\pi_2$. The heat released at C_1 at temperature $T+\left(\frac{\Delta T}{2}\right)$ is $q\sigma_{c1}\Delta T$ and heat absorbed in metal C_2 at temperature $\left(T+\frac{\Delta T}{2}\right)$ is $q\sigma_{c2}\Delta T$.



$$\frac{q\pi_1}{T+\Delta T} - \frac{q\pi_2}{T} - \frac{q\sigma_{c1}\Delta T}{T+\left(\frac{\Delta T}{2}\right)} + \frac{q\sigma_{c2}\Delta T}{T+\left(\frac{\Delta T}{2}\right)} = 0$$

$$\pi = T\frac{dE}{dT}$$

Materials for thermoemf sensors :

Material choice is guided by :

1) high thermoemf per unit temperature change, that is thermoelectric power.
2) low electrical resistance of the couple.
3) Linearity of E-T curve over the range of interest.
4) High melting point of the couple materials for wider range.

Nonmetallic thermocouples have been proposed to be used in atmospheres containing carbon, with operating voltages and different temperatures about 2200°C.

E-T Relations :

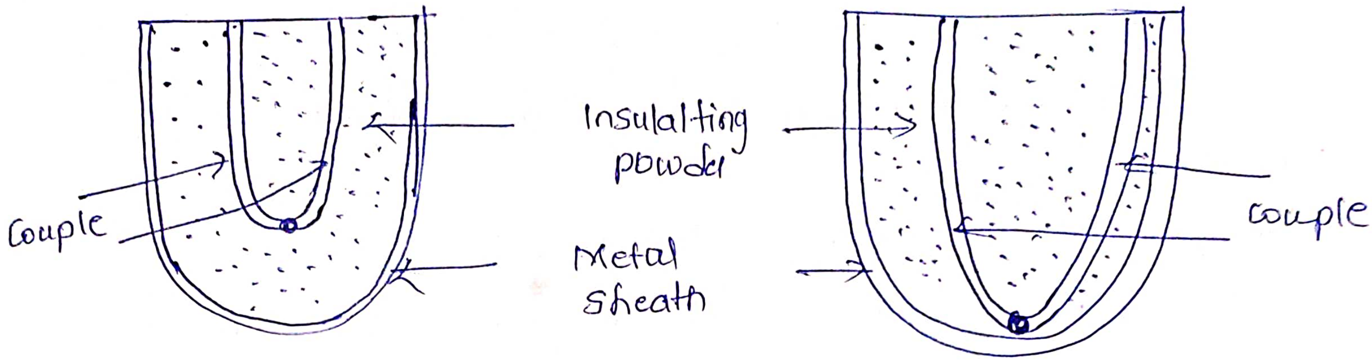
The thermocouple emf output can be expressed as a series function of temperature t.

$$E = \sum_{j=0}^{n} a_j t^j$$

Construction:

construction of a couple complete with protection varies usually, the kept separated by small insulator beads (single-hole, twin-hole, 4-hole type are common). The entire thermocouple with such insulator sleevings is now enclosed in a porous ceramic tube and finally enclosed in a metal sheath.

Thermocouple type	Sheath Material
J	Stainless steel (Ni(8)), Cr(18), Fe)
K, N	Inconel (Ni (trace), Cr(15), Fe)
R, S.	Stainless steel), Inconel
G	Molybdenum - tungsten steel



The MI thermocouples.

Reference Temperature :

This is often referred as the 'Cold junction' temperature in temperature sensing above. $0°C$. A metal block is heated and maintained at a temperature by thermostatic control and the insulated reference junction is attached to it.

$$t_c = t_M + K t_R$$

t_c is correct temperature
t_m is measured temperature
t_R is reference temperature

K is the ratio of thermoelectric powers at t_M and t_R.

Thermo-sensors using semiconductor Devices :

The working of semiconductor thermosensors is explained by

$$\Delta E = \alpha_s \Delta T$$

ΔE is open circuit emf
α_s is Seeback coefficient
ΔT is difference of temperature between two junctions

$$\alpha_s = \frac{\lambda K}{T} \ln(P/P_o)$$

λ is constant value of 2.6
P_o is about $5 \times 10^{-6} \Omega m$.

* Junction Semiconduction Types:

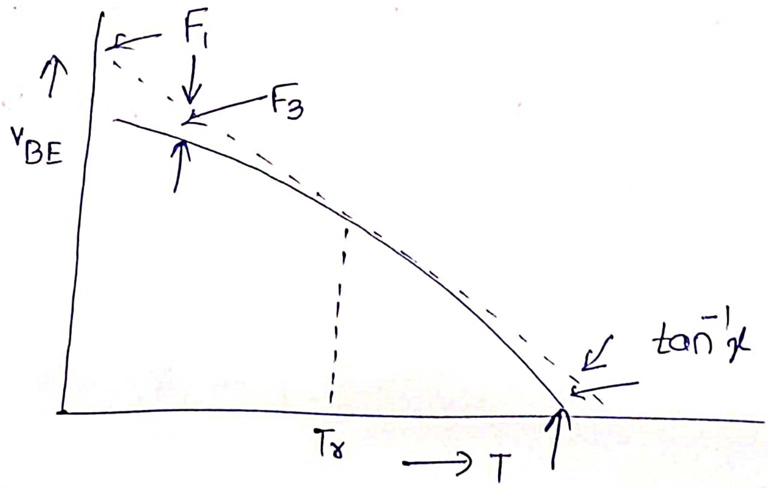
Junction semiconductor (Si, GaAs, Ge) diodes and transistors have their base-emitter voltage V_{BE} related to temperature T, with a normal range of 1K - 200°C as usable range and covers an overall range of -50° - 150°C for good linearity.

The transistor in a conducting state, the voltage has a temperature coefficient of value $-2\ mV/°C$. then the relation between V_{BE} and T is given by

$$V_{BE} = \frac{KT}{q} \ln\left(\frac{I}{I_s}\right)$$

q is electron charge
K is Boltzmann constant,
I_s is saturation current

$$\frac{K}{q} = 86.17\ \frac{\mu V}{K}$$

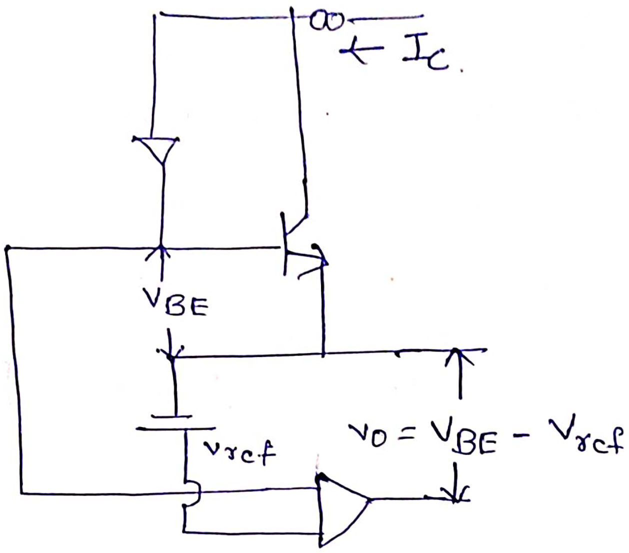


V_{BE} - T curve of a semiconductor diode temperature sensor with linear approximation value.

The non-linearity appearing due to the third term is temperature-dependent as also dependent on $(m_1 - m_2)$, for small $\Delta T = T_1 - T_r << T_r$.

A single transistor sensor works at a low cost accurate linear indication purpose, it requires constant current source I_c, for this the $m_2 = 0$ is simplified to

$$V_o = V_{BE1} - V_{BE2} = K_d (T_1 - T_2)$$



This single transistor sensor with these current and voltage biasing indicates that the differential temperature replaces V_{ref} with a second sensor of output V_{BE2} and K_d is a constant value.

The PTAT Sensor :

The single transistor sensor is extended into an IC temperature transducer which includes biasing, amplifying, linearizing circuits. Such sensor is called PTAT sensor Proportional To the Absolute Temperature. where

$$\Delta V_{BE} = \frac{KT}{q} \ln\left[\frac{I_{C1} I_{S2}}{I_{C2} I_{S1}}\right]$$

Here I_S is proportional to emitter area and the equation becomes.

$$\Delta V_{BE} = \frac{kT}{q} \ln(\gamma_1 \gamma_2)$$

$$\gamma_1 = \frac{I_{S2}}{I_{S1}}, \quad \gamma_2 = \frac{I_{C1}}{I_{C2}}$$

* Thermal Radiation sensors :

Most of the thermal sensors requires 'quantum' of heat by conduction, whereas the thermal radiation sensors requires no physical contact, these sensors are guided by basic laws of black body radiation such as Planck's law and Stefan - Boltzmann law.

These laws states that "the radiation flux emitted by a black body per unit solid angle per unit area in a direction normal to it in the wavelength range λ to $\lambda + d\lambda$ is given as $L_\lambda d\lambda$. where L_λ is spectral radiance

$$L_\lambda = \frac{C_1}{\lambda^5 \exp\left[\frac{C_2}{\lambda T}\right]}$$

C_1 & C_2 are constants, $C_1 = 2hc^2 = 3.742 \times 10^{-16}\ \mathrm{Wm^2}$.

$C_2 = \frac{hc}{K} = 1.4388 \times 10^{-2}\ \mathrm{mK}$.

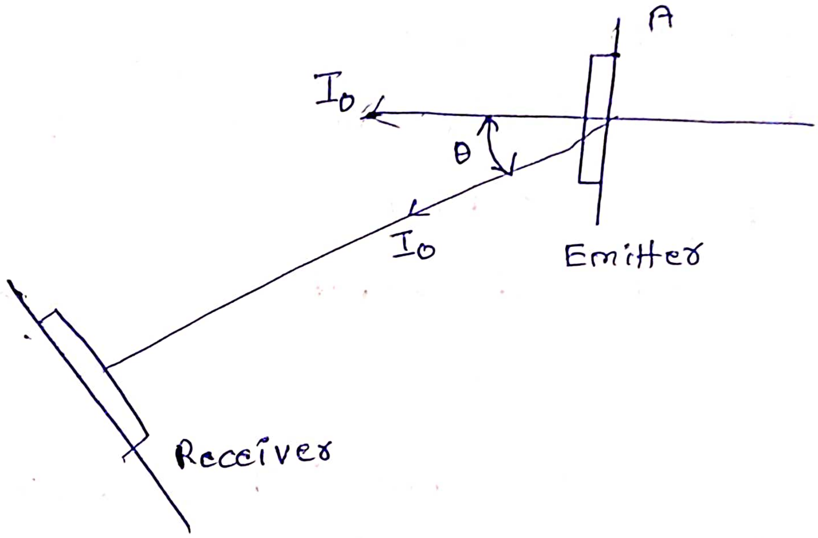
Planck deduced the equation to

$$L_\lambda = \frac{C_1 \mu^2}{\lambda^5 (e^{C_2/\lambda T} - 1)}$$

μ is the refractive index of the medium, unity for air.

The flux emitted by a unit solid angle by a source is called intensity $i = \frac{d\phi}{d\omega}$, expressed in watts per steradian, the flux emitted by a surface is called radiant excitance M

$M = \frac{d\phi}{dA}$. Radiance L, the flux per unit area per unit solid angle, is given by $L = \frac{d^2\phi}{(dA \cos\phi \, d\omega)}$ is shown in



Radiation emission and reception at angles other than perpendicular to the source

The total radiance L of a black body is obtained by integrating L_λ over the range of λ.

$$L = \int_0^\infty L_\lambda \, d\lambda = \frac{\mu^2 \sigma T^2}{\pi}$$

which, in terms of M

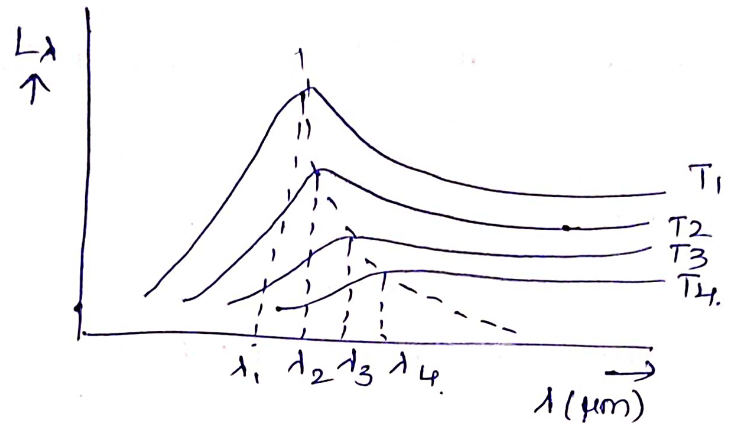
$$M = \pi L = \sigma T^4 \quad \text{for } \mu = 1.$$

σ is the stefan-Boltzmann constant and has a value $5.67 \times 10^{-8} \, Wm^{-2}K^{-4}$.

$$\sigma = \frac{2\pi^5 K^4}{15 h^3 c^2}$$

for different values of T

$$\lambda_i T_i = \lambda_n T_n = \lambda_m T_m = 2898\,\mu m\,k.$$



$L_\lambda - \lambda$ curves for varying temperatures.

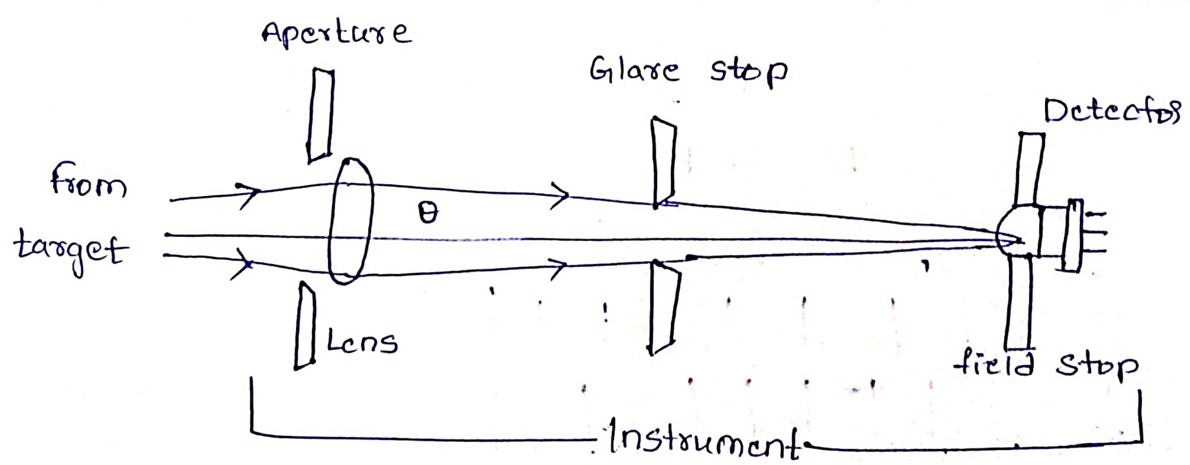
When the surface roughness increases or the degree of oxidation on the surface increases, emittivity increases.

Radiation thermometers are broadly classified into :

(i) Total Radiation type
(ii) Multiwaveband type
(iii) Single waveband type or spectral radiation type
(iv) Ratio type.

The most commonly used type uses the equation.

$$E = \pi \tau L \sin^2\theta$$ [θ is field cone half angle]



* Detectors :

All total radiation thermometers use a few 'standard' detectors. works on wavelength/frequency selective in nature. The factors that are taken into account while selecting a detector are

a) Responsivity.
b) Spectral range coverage.
c) noise equivalent power (NEP)
d) Speed of response,
e) linearity
f) stability
g) Operating temperature
h) operating mode.

The detectivity is defined as

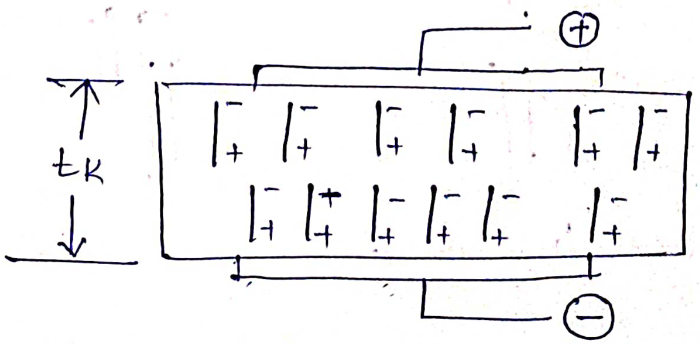
$$D_\lambda = \left[\frac{1}{NEP}\right]\sqrt{\left(\frac{A\Delta\omega}{2A}\right)} \; \mathrm{CmHz}^{1/2}\,W^{-1}.$$

A is detector Area
$\Delta\omega$ is frequency bandwidth

* The pyroelectric Thermal sensors

These are comparitively new and comprises of a ferroelectric material. The direction of polarization can be changed by the applied electric field.

$$P = \sigma E$$



* Quartz crystal Thermoelectric sensors :

Single crystal SiO_2, known as quartz is commonly available with a modification as α-quartz consisting of three SiO_2 molecules in its elementary cell, and its axis of symmetry is called the optic axis, the plane of Polarization of light rotates anticlockwise or clockwise and accordingly quartz is named left or right.

It has elastic, piezoelectric and resonating properties these are important to make this quartz as a sensor. It has low acoustic and good chemical properties.

The propagation of elastic waves are given by

$$\rho v^2 \varepsilon_j = \sum_{i=1}^{3} \psi_{ji} \varepsilon_i$$

ε = particle elongation
ρ = density of material
v = velocity of propagation.
ψ = elastic stiffness

$$\psi_{ji} = \sum_{p=1}^{3} \sum_{q=1}^{3} C_{jpiq} \alpha_p \alpha_q.$$

α is the direction cosines of propagation w.r.t crystal axis

$$\begin{vmatrix} \psi_{11} - \rho v^2 & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} - \rho v^2 & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} - \rho v^2 \end{vmatrix} = 0$$

The resonance frequency is given by

$$f_{nr} = \frac{n}{2h}\sqrt{\frac{C_r'}{\rho}} \left[= \frac{n v_r}{2h} \right]$$

All these factors depend on temperature as does f. A truncated polynomial representing the f-T relation is

$$\frac{f(T) - f(T_0)}{f(T_0)} = \sum_{j=1}^{3} \alpha_{fj} (T - T_0)^j$$

T_0 is reference temperature

α_{ji} is j^{th} order temperature coefficient of frequency.

$$\alpha_{fj} = \frac{1}{f_0} \frac{1}{j!} \frac{\partial^j f}{\partial T^j}$$

These coefficients are also dependent on cuts of the crystal.

Typical values are

$$\alpha_{f1} = 9 \times 10^{-5}\, K^{-1} \quad \text{with} \quad \frac{\alpha_{f1}}{\alpha_{f2}} = 1.5 \times 10^{+3}\, K.$$

$$\frac{\alpha_{f1}}{\alpha_{f3}} = 3 \times 10^{6}\, K$$

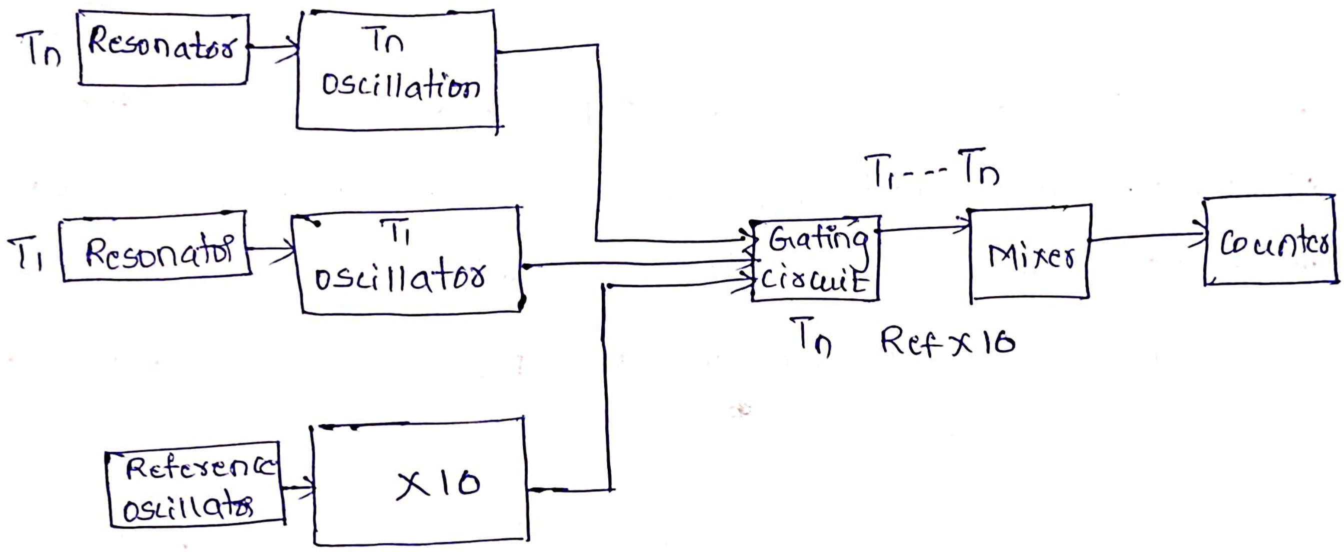
The crystals so cut are used as resonators with the resonance frequency varying with temperature, the change in

$\frac{\{f(T) - f(T_0)\}}{(T - T_0)}$ slope is given by $f(T_0)\alpha_{f1} = f(T_0) \times 4 \times 10^{-5}\, Hz/K.$

for a resonant frequency of 20 MHz at $T_0 = 273K$, the slope becomes 800 Hz/K.

The resonator is often in the form of a tuning fork, it is coupled to an oscillator producing a base frequency $f(T_0)$. The frequency is measured by counting 'pulses' in

$$t_{int} = \frac{N}{f(T_0)\left[1 + \sum_{j=1}^{3} \alpha_{fj} (T_0 - T)^j\right]}$$



Basic scheme of an n-channel crystal resonator type temperature meter

* NQR Thermometry

The basic principle of Nuclear Quadrupole Resonance (NQR) thermometer is that the NQR frequency of a certain nuclei varies with temperature.

Ex : ^{35}Cl in $KClO_3$.

The nucleus possesses an electric quadrupole moment which interacts with electric field gradients generated by surrounding ions in compound lattice and valence electrons of nucleus.

The frequency corresponds to the separation of energy of two components is given by

$$\nu_0 = \frac{eQ\dot{E}_a}{2h}$$

e is electron charge
Q is electrical Quadrupole moment
$\dot{E}_a$ is electric field gradient tensor component along the principle axis

This is the frequency near 0 K and is seen independent of temperature, but with rising temperature, for example in $KClO_3$, ClO_3 ion has torsional vibrations and there occurs fluctuation in the orientation of $\dot{E}_a$ and the resonant frequency ν is given by.

$$\nu = \nu_0 \left[1 - \sum_i \frac{3M_{ri} h}{4\pi\omega_i} \left\{ \frac{1}{2} + \frac{1}{\exp\left(\frac{h\omega_i}{2\pi KT - 1}\right)} \right\} \right]$$

upto about 50 – 60 K.

M_{ri} is reciprocal of the moment of inertia of the i^{th} lattice
ω_i is angular frequency of i^{th} node of vibration of lattice.

At higher temperatures the lattice expands and the equation is modified for the range 60 – 300K, as

$$\nu = \nu_0 \left[1 - \frac{3K}{2M_{ri}\omega_i} \left\{ \sum_i \frac{h\omega_i/(2\pi K)}{\exp(h\omega_i/2\pi KT) - 1} \right\} \right] - q(T)$$

$q(T)$ is quartic polynomial in T.

* Spectroscopic Thermometry :

For special cases of temperature measurement in heated gases, plasma, flames and stellar objects, spectroscopic techniques were used, these are different depending on the media and also in the way they can be probed.

In some cases, atomic, ionic or molecular spectral lines or bands are produced which have intensities that vary with temperature. Intensities of atomic, ionic and molecular lines are measured only relatively and the relation is:

$$\frac{I_1}{I_0} = \frac{A_1}{A_0}\left[\exp\left(-\frac{E_1 - E_0}{KT}\right)\right]$$

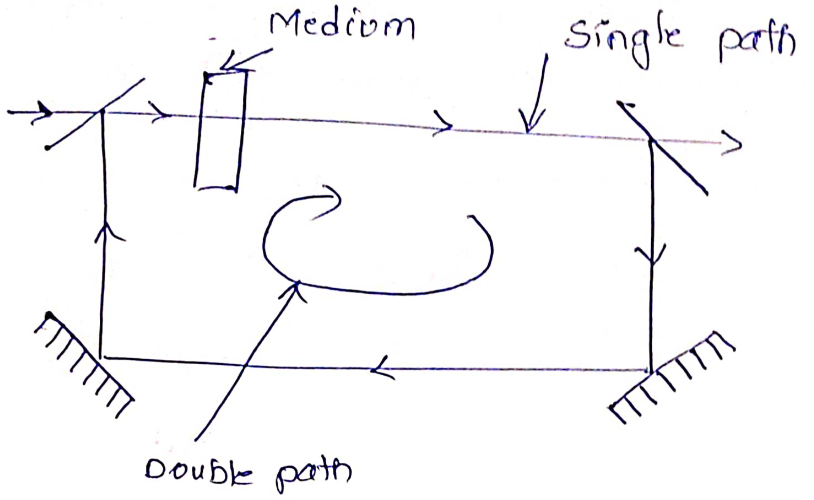
I_1 is intensity of the line to be measured.
I_0 is intensity of reference line
E_1 is energy of level before the change.

from the above equation T can be measured by

$$T = \frac{E_1/K}{\left\{c - \ln\left(\frac{I_1}{A_1 v_1^4}\right)\right\}}$$

c is constant dependent on energy, intensity and frequency of reference line and v_1 is the frequency of measured line.

The spectral radiance from a standard source is spectroscopically viewed through the measured medium, this source is a calibrated variable temperature source. The spectral line arising in the medium would appear brighter or darker depending on whether the temperature is lower or higher than the medium temperature, the correct temperature is obtained when the reversal occurs.



Comparison of spectral radiances of two beams arising from the same source - one moving along a single path through the medium, the other crossing the medium twice obtained by using a mirror as shown in above figure. The temperature of the medium is calculated through Planck's Law where the increased thermal motion of radiation particles due to increasing temperature of the gaseous medium. The width of the line $\Delta\lambda_w$ is related to temperature as

$$T = \frac{M \Delta\lambda_w^2}{2.05\lambda^2} \times 10^{12}$$

M is the molar mass, $10^{12}/2.05$ is made up of a gas constant and velocity of light.

Lase induced scattering is also used to measure temperature and the coherent anti-stokes Raman Spectroscopy (CARS) is suitable, and requires three laser beams, two of identifal frequencies, ν_1 and the other of tunable lower frequency ν_2. The three beams intersect at the medium and when $\nu_1 \sim \nu_2$ is around the Raman active resonance frequency, a coherent anti-Stokes spectrum is generated $(2\nu_1 - \nu_2)$, by measuring line width, temperature can be calullated.

* Noise Thermometry :

These are basically metal conductors in which random statistical thermal agitation of the elections in the conduction band is measured following a theamodynamic relationship proposed by Nyquist.

It states that with temperature above 0K, a voltage fluctuating statistically around zero is obtained across a passive element / network and its mean square value $\Delta\bar{V}^2$ is given by.

$$\Delta\bar{V}^2 = 4kT \cdot Re\{Z(f)\}\,\Delta f$$

$Re\{Z(f)\}$ is real part of the complex impedance $Z(f)$ of element/network

f is frequency.

T is absolute temperature,

K is Boltzmann constant.

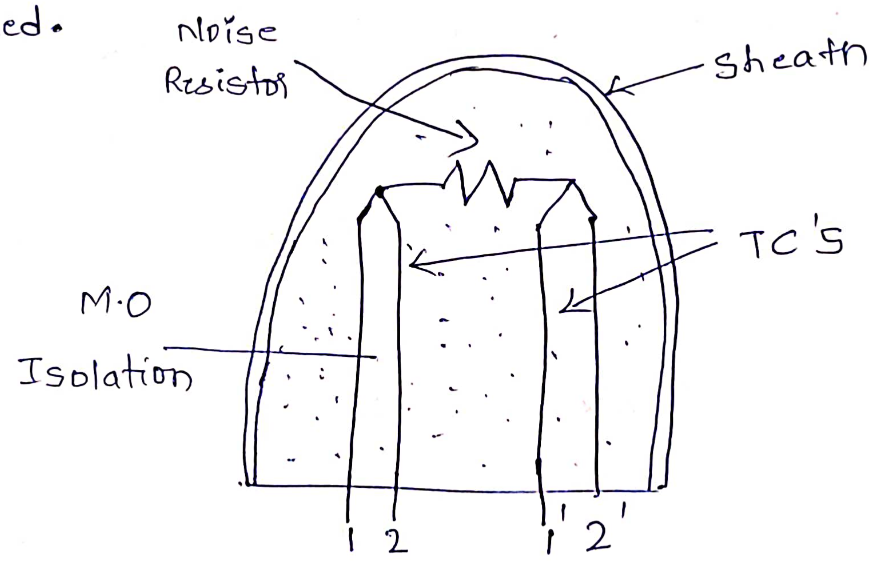
The power spectrum of thermal noise P_s is given by

$$P_s = \frac{\Delta\bar{V}^2}{\Delta f} = 4kT\,Re\{Z(f)\}$$

The thermal noise is independent of the chemical composition, physical state of the substance, nature of charge carriers. Hence all elements are suitable for making sensors. But it is dependent on the resistance R which is affected by environmental changes, because of this influences the noise thermometer shows no change.

The disadvantage with noise thermometer is its very low output, for a reasonable value of R and frequency band of 0.1 MHz. the output is less than a microvolt.

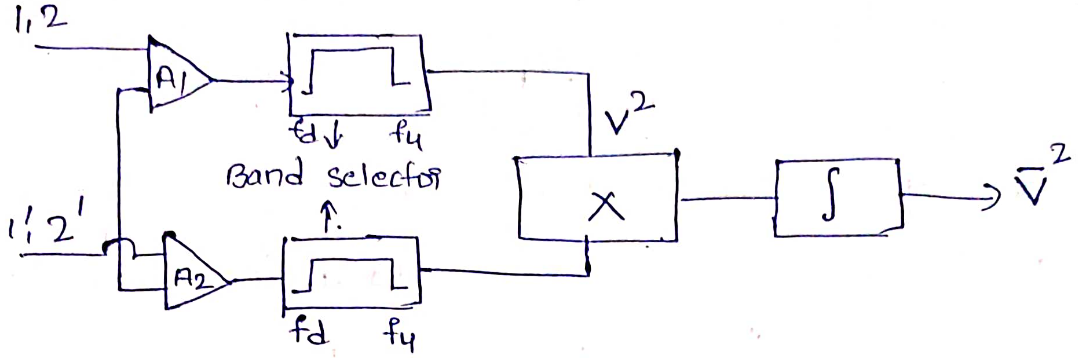
In order to overcome this, a combined thermometer (CT), using a thermocouple (TC) and a noise thermometer (NT) is designed.



Scheme of a TC-NT Sensor.

It consists of a pair of sheathed thermocouples with two hot junctions and a noise resistor between them. Each individual couple measure the temperature in usual fashion, the noise resistor has 4 lead wires, while measuring noise temperature, they can be arranged to eliminate the resistance of these wires.

by cross correlation. Since noise temperature is fluctuating in nature, it can be separated out from ITC dc thermocouple output by using a capacitor. The scheme of measurement is shown in

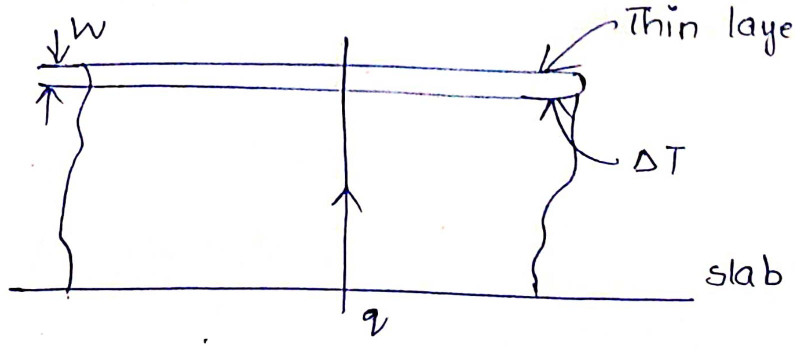


As the noise voltage is stochastic in nature, measurement is susceptible to statistical errors. For the measurement bandwidth $\Delta f = f_u - f_d$ and the measurement time t_M, the relative error is calculated as

$$E_R = \frac{\Delta T}{T} \sim (\Delta f t_M)^{-1/2}$$

* Heat Flux Sensors :

In some cases like, there needs a measurement of total heat flow or heat flux needs to be measured, where the heat is transmitted through a wall, specific heat, heat of melting or solidification, heat of hydration, heat of reaction and so on.. The principle of heat flux measurement is dependent on measurement of temperature difference across a thin layer added to the slab of a 'homogeneous' material through which the heat is transmitted.



The above figure shows the scheme of heat flux transmission, if q is the heat flux, ΔT is the temperature difference, and λ is the thermal conductivity of the material of the thin layer of thickness w, then

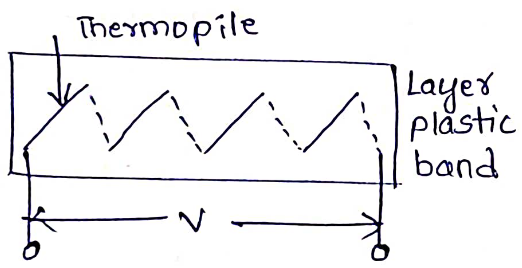
$$q = \left(\frac{\lambda}{w}\right) \Delta T$$

One of the types of heat flux sensors consists of serially connected thermopiles over or embedded in a thin layer of rubber or plastic. If n is the number of thermocouples forming the thermopile, ΔT is the temperature difference across the layer, and E is the thermoelectric power of the each thermocouple, then the thermopile output voltage V is given as

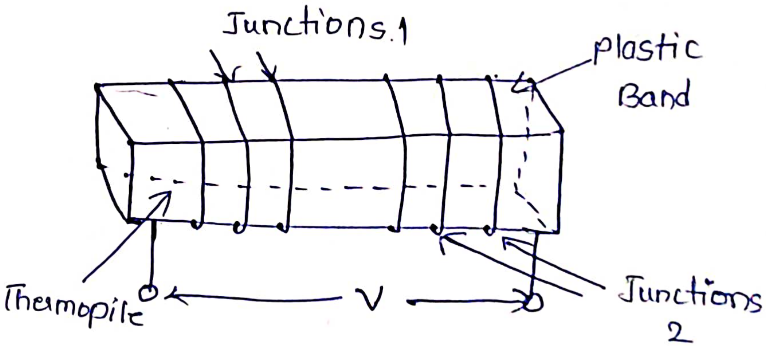
$$V = nE\Delta T$$

$$q = \left(\frac{\lambda V}{wnE}\right) = K_1 V$$

Examples:



Embedded type heat flux Sensor



half-plated wound-wire type heat flux sensors

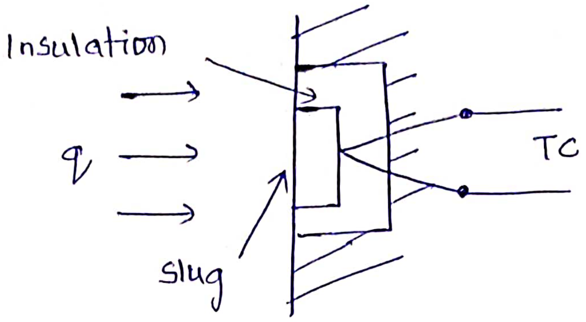
Thermopiles are made using more recent techniques rather than winding dissimilar metal wires.

For measurement of heat flow rate across a surface, often a metal slug is embedded in it. If mass is m, a is surface area, c is the specific heat of slug, and temperature rise is ΔT measured by a thermocouple and then heat transfer rate q

is given by.

$$aq\,dt = mC\,dT$$

$$(cm^2)(W/cm^2)(s) = (kg)(Ws/kg^{\circ}C)(^{\circ}C)$$



Heat sensor using slug technique.

However, heat losses are considered, an additional term for heat transfer rate due to heat loss given by $K_2(t_{s1} - t_{ca})$ is taken, where K_2 is the loss coefficient, t_{s1} is slug temperature and t_{ca} is the casing temperature. Hence.

$$q = \left(\frac{mC}{a}\right)\left(\frac{dT}{dt}\right) + K_2\,(t_{s1} - t_{ca})$$

X